Erratum to "Distinction for unipotent p-adic groups"

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Abstract

The Kirillov classification in [4] is misstated, namely [4, Theorem 3.6, 3)] is incorrect. We correct it here, as well as the incorrect proof of [4, Theorem 5.2] which follows fromthis misstatement. One can also check https://arxiv.org/abs/1909.07289v5 for a full corrected version.

First [4, Lemma 4.3] and the discussion before it must be repalced by:

We make σ act on $\mathcal{N}^*$ by the formula

$$\sigma(\phi) = -\phi^\sigma.$$

Then a very special case of [4, Lemma 4.2] is:

Lemma 0.1. Take $\phi \in \mathcal{N}^*$, then $\sigma(\phi)$ and ϕ are in the same U-orbit if and only if there is a σ-fixed linear form in the U-orbit of ϕ, i.e. a linear form which vanishes on $\mathcal{N}^\sigma$.

Then [4, Theorem 3.6, 3)] should be replaced by the following statement:

3) Two irreducible representations $\pi(\mathrm{U}', \mathrm{U}, \psi_\phi)$ and $\pi(\mathrm{U}'', \mathrm{U}, \psi_{\phi'})$ are isomorphic if and only if ϕ and ϕ' are in the same U-orbit for the co-adjoint action.

The last two sentences of the proof of [4, Theorem 3.6, 3)] should be replaced by:

By induction this means that $\phi_{|\mathcal{N}_0}$ and $\phi'_{|\mathcal{N}_0}$ are U_0-conjugate. Then it is explained just before [3, Lemma 5.2] at the end of the proof of [3, Theorem 5.2] that this implies that ϕ and ϕ' are indeed U-conjugate.

One can then introduce the following notation:

Notation 0.2. The isomorphism class of the irreducible representation $\pi(\mathrm{U}', \mathrm{U}, \phi)$ only depends on ϕ, we set

$$\pi(\psi_\phi) := \pi(\mathrm{U}', \mathrm{U}, \psi_\phi).$$

Finally the proofs and statements of [4, Theorem 5.2 and Corollary 5.3] must be corrected as follow, thanks to Lemma 0.3 hereunder:

Note that $(\mathcal{N}^*)^\sigma$ and $(\frac{\mathcal{N}}{\mathcal{N}^\sigma})^*$ are canonically isomorphic, and we identify them. It is a space acted upon by U^σ. Before stating the main theorem, we recall [1, Lemma 2.2.1], the proof of which is valid over F (as it relies on [2, Proposition 1.1.2] which has no assumption on the field).

Lemma 0.3. Take $\phi \in (\frac{\mathcal{N}}{\mathcal{N}^\sigma})^*$, then there is a σ-stable Lie sub-algebra $\mathcal{N}'$ of $\mathcal{N}$ such that $(\phi, \mathcal{N}')$ is polarized.

We can now prove the following result.

Theorem 0.4. A representation $\pi \in \mathrm{Irr}_{\mathrm{U}^\sigma}(\mathrm{U})$ is distinguished if and only $\pi^\vee = \pi^\sigma$. Moreover the map $\mathrm{U}^\sigma.\phi \mapsto \pi(\psi_\phi)$ is a bijection from $\mathrm{U}^\sigma \backslash (\frac{\mathcal{N}}{\mathcal{N}^\sigma})^*$ to $\mathrm{Irr}_{\mathrm{U}^\sigma\text{-dist}}(\mathrm{U})$.

Proof. Suppose that $\pi = \pi(\psi_\phi) \in \mathrm{Irr}(\mathrm{U})$ is conjugate self-dual, then $\sigma(\phi)$ and ϕ' are in the same U-orbit, which must contain a σ-fixed linear form thanks to Lemma 0.1. So we can in fact suppose that $\phi \in (\frac{\mathcal{N}}{\mathcal{N}^\sigma})^*$. In particular by Lemma 0.3 we can write $\pi(\psi_\phi) = \pi(\mathrm{U}', \mathrm{U}, \psi_\phi)$ for $\mathrm{U}' = \exp(\mathcal{N}')$ which is σ-stable. The quotient $\mathrm{U}'^\sigma \backslash \mathrm{U}^\sigma$ identifies with a closed subset of $\mathrm{U}' \backslash \mathrm{U}$ and the condition $\phi \in (\frac{\mathcal{N}}{\mathcal{N}^\sigma})^*$ implies that ψ_ϕ is trivial on U'^σ. Then π is distinguished, with explicit linear nonzero U^σ-invariant linear form given on π by

$$\lambda : f \mapsto \int_{\mathrm{U}'^\sigma \backslash \mathrm{U}^\sigma} f(u) du.$$

To finish the proof it remains to prove the injectivity of the map $\mathrm{U}^\sigma.\phi \mapsto \pi(\psi_\phi)$, which is [4, Lemma 4.4]. □

In particular in the case of the Galois involution one gets a bijective correspondence between $\mathrm{Irr}(\mathrm{U}^\sigma)$ and $\mathrm{Irr}_{\mathrm{U}^\sigma\text{-dist}}(\mathrm{U})$. Indeed $\mathbf{U} = \mathrm{Res}_{E/F}(\mathbf{U}^\sigma)$ for E a quadratic extension of F. Writing δ for an element of $E - F$ with square in F. One can identify the space $(\mathcal{N}^\sigma)^*$ to the space $(\mathcal{N}^*)^\sigma$ by the map

$$\mathrm{C} : \phi_\sigma \to \phi$$

where

$$\phi(N + \delta N') = \phi_\sigma(N').$$

This yields:

Corollary 0.5. When E/F is a Galois involution, the map $\pi(\psi_{\phi_\sigma}) \to \pi(\psi_\phi)$ is a bijective correspondence from $\mathrm{Irr}(\mathrm{U}^\sigma)$ to $\mathrm{Irr}_{\mathrm{U}^\sigma\text{-dist}}(\mathrm{U})$

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References

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